

Summary: Linear Least Squares (Chapter 8)

Basic: $y = v_1 + v_2x$

Error Function:

$$E(v_1, v_2) = \sum_{i=1}^n (v_1 + v_2x_i - y_i)^2 \quad (8.9)$$

Solution:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{n \sum x_i^2 - (\sum x_i)^2} \begin{pmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{pmatrix} \begin{pmatrix} \sum y_i \\ \sum x_i y_i \end{pmatrix} \quad (8.11)$$

Error Function:

$$E(v_1, v_2) = \sum_{i=1}^n \left(\frac{v_1 + v_2x_i - y_i}{y_i} \right)^2 \quad (\text{Exercise 8.17})$$

Solution:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{ad - b^2} \begin{pmatrix} a & -b \\ -b & d \end{pmatrix} \begin{pmatrix} \sum 1/y_i \\ \sum x_i/y_i \end{pmatrix}$$

where $a = \sum x_i^2/y_i^2$, $b = \sum x_i/y_i^2$, and $d = \sum 1/y_i^2$

General

Error Function:

$$E(\mathbf{v}) = \|\mathbf{A}\mathbf{v} - \mathbf{y}\|_2^2 \quad (8.12)$$

Moore-Penrose Solution:

$$\mathbf{v} = \mathbf{A}^+\mathbf{y}$$

where

$$\mathbf{A}^+ \equiv (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T \quad (8.25)$$

QR Solution (GS version):

$$\mathbf{R}_m\mathbf{v} = \mathbf{y}_m \quad (8.22)$$

where $\mathbf{R}_m = \mathbf{G}^T\mathbf{A}$, $\mathbf{y}_m = \mathbf{G}^T\mathbf{y}$, and $\mathbf{G} = \text{GS}(\mathbf{A})$

SVD Solution: $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$

$$\mathbf{A}^+ = \mathbf{V}\mathbf{\Sigma}^{-T}\mathbf{U}^T \quad (\text{Exercise 8.7})$$